Gradient-index filters: conversion into a two-index solution by taking into account dispersion

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The possibility of converting the continuous refractive-index profile into a two-index solution is shown. The technique of conversion that is developed makes it possible to take dispersion into account. This ensures that it is possible to achieve a good agreement between theory and practice in a broad spectral range. Several gradient-index filters have been developed and produced, and measuring results are presented.

Key words: Gradient-index filters, inverse Fourier transformation, dispersion, two-index solution, synthetic dispersion, initial correction.

Introduction

A limited number of papers have been published dealing with the subject of how to design gradient-index filters (GIF's) by using an inverse Fourier transformation technique.[1–6] However, until now research has been restricted to theoretical aspects, and the types of thin-film component that could be designed have been limited.

It is the aim of Lys & Optik to move gradient-index technology toward practical use and greater flexibility. Recently a new correction method was presented that makes it possible to design GIF's with steep skirts and high reflection.[7] One advantage of the gradient-index technique is that it is possible to design a thin film with a broadband spectral performance that can hardly be achieved by means of classical thin-film techniques. It is, however, a problem that the original theory does not take dispersion into account. The scope of this second paper is to show that it is possible to take dispersion into account and to convert the solution into a system of layers, which can actually be produced by standard vacuum techniques.

Why Convert?

In principle it should be possible to vary the refractive index of a growing layer within some limits by the codeposition of one low-index material and one high-index material. However, it is rather complicated to measure the refractive index in the top layer of a continuously growing gradient-index structure that is vacuum deposited inside a vibrating and rotating chamber system. Using a process ellipsometer may be a solution. However, we believe that it would be extremely difficult, if not impossible, to achieve the necessary stability and resolution in an ellipsometric measuring system.

Conversion of the continuous gradient-index profile into a two-index solution consisting of layers that are thin compared with the wavelength of light appears to be another solution to the problem. At the same time conversion into a two-index solution makes it possible to take into account dispersion and the nonperpendicular incidence of light.

A gradient-index coating design using thin high- and low-index layers has been presented by Southwell.[8] However, by using the inverse Fourier technique one degree of freedom is reestablished in the design, because the thicknesses of the different layers are not fixed. At the same time the randomlike optimization process is replaced by a Fourier-based closed-loop optimization process.

Converting the Refractive-Index Profile

The transmittance curve is calculated on the basis of N numbers describing the refractive-index profile. Each number represents the center value of the refractive index in a thin slice of the inhomogeneous layer. When calculating the transmission curve by means of the matrix method,[9] we treat each layer as if

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it were homogeneous with some fixed optical thickness OT. In this way the GIF is always treated as if it were a multilayer system consisting of thin layers.

The characteristic matrix for a homogeneous layer of index $n(x)$ and optical thickness OT is (normal incidence)

$$\mathbf{M} = \begin{bmatrix} \cos \Phi & i \sin \Phi / n(x) \\ i\, n(x) \sin \Phi & \cos \Phi \end{bmatrix}, \tag{1}$$

where Φ is the phase, which is given by

$$\Phi = 2\pi/\lambda\, OT = 2\pi/\lambda\, n(x) t(x), \tag{2}$$

where λ is the wavelength of the incident light and $t(x)$ is the physical thickness of the layer, which can expressed as

$$t(x) = OT_{\mathrm{tot}}/[(N - 1) n(x)], \tag{3}$$

where OT_{tot} is the total optical thickness of the structure.

By choosing some resolution that satisfies

$$OT = n(x) t(x) \ll \lambda, \tag{4}$$

we may replace the trigonometric functions in Eq. (1) by their small argument approximations:

$$\mathbf{M} = \begin{bmatrix} 1 & i\, 2\pi/\lambda\, t(x) \\ i\, 2\pi/\lambda\, n^2(x) t(x) & 1 \end{bmatrix}. \tag{5}$$

Southwell[8] has shown that it is possible to convert such a thin layer into a combination of a thin layer of high refractive index n_H and a thin layer of low refractive index n_L.

The characteristic matrix of such a two-layer equivalent is found by matrix multiplication of its respective characteristic matrices and dropping second-order terms in thickness:

$$\mathbf{M} = \begin{bmatrix} 1 & i\, 2\pi/\lambda [t_H(x) + t_L(x)] \\ i\, 2\pi/\lambda [n_H^2 t_H(x) + n_L^2 t_L(x)] & 1 \end{bmatrix}. \tag{6}$$

By comparing Eqs. (5) and (6), it is easy to derive equations describing the physical layer thicknesses, $t_L(x)$ and $t_H(x)$:

$$t_H(x) = \frac{n^2(x) - n_L^2}{n_H^2 - n_L^2} t(x), \tag{7}$$

$$t_L(x) = t(x) - t_H(x). \tag{8}$$

From Eq. (6) it is seen that it does not matter in which order the two layers appear. By converting all uneven indexed sublayers into HL equivalents and all even indexed sublayers into LH equivalents, it is possible to convert the continuous refractive-index profile into a two-index solution consisting of $(N + 1)$ sublayers. Deviations between the achieved spectral characteristic of the converted filter and the transmission curve before conversion can be reduced by means of successive approximations.[7] However the degree of agreement between the desired and the obtained spectral characteristic depends on the resolution N in the calculations. This is comparable with what is known from the digital theory (the Nyquist criteria).

Taking Dispersion into Account

One advantage of the gradient-index technique is that it is possible to design a thin film with a broadband spectral performance that can hardly be achieved by means of classical thin-film techniques. It is, however, a problem that the original theory[1–3] does not take dispersion into account. In the following we show that the developed technique of conversion makes it possible to introduce some synthetic dispersion within the design stage.

Imagine that we have chosen the materials to be used for the production of the designed filter and that the dispersion of these materials, $n_L(\lambda)$ and $n_H(\lambda)$, is well characterized. In this case it is possible to compare each subpart of the refractive-index profile with a pair of layers if one chooses some wavelength of conversion λ_c:

$$t_H(x) = \frac{n^2(x) - n_L^2(\lambda_c)}{n_H^2(\lambda_c) - n_L^2(\lambda_c)} t(x), \tag{9}$$

$$t_L(x) = t(x) - t_H(x). \tag{10}$$

It is now possible to have an expression describing the refractive-index profile including some synthetic dispersion:

$$n(x, \lambda) = \{n_H^2(\lambda)[t_H(x)/t(x)] + n_L^2(\lambda)[t_L(x)/t(x)]\}^{1/2}. \tag{11}$$

By comparing Eqs. (9)–(11) it is possible to rewrite Eq. (11):

$$n(x, \lambda) = [A(x) n_H^2(\lambda) + [1 - A(x)] n_L^2(\lambda)]^{1/2}, \tag{12}$$

where

$$A(x) = \frac{n^2(x) - n_L^2(\lambda_c)}{n_H^2(\lambda_c) - n_L^2(\lambda_c)}. \tag{13}$$

By converting the continuous refractive-index profile into a two-index solution, it appears to be possible to include dispersion, if the dispersion of the two materials is known. By using matrix multiplications, one obtains a transmission curve that corresponds to the dispersion-corrected refractive-index profile [Eq. (11)].

The transmission curve obtained differs more or less from the transmission curve desired because of the synthetic dispersion. This is because dispersion is still not included within the Fourier calculations themselves. The synthetic dispersion only helps to obtain an impression of which deviations would be present if one were to convert the refractive-index profile at this point.

If the deviations are not too large, it is possible to reduce them by means of successive approximations as described in Ref. 7. Finally, when the refractive-index profile is converted into a real two-index solution, one sees that the solution is actually dispersion corrected. However, in the following an important initial correction method is presented that makes the dispersion-dependent calculations much more efficient.

Dispersion-Related Initial Corrections

Imagine that a filter has been designed without taking dispersion into account (see the dotted curve in Fig. 1) and that the refractive indices correspond to the real ones at the wavelength λ_0 (500 nm here). In this case one sees that the transmission curve is shifted when dispersion is included (see the solid curve in Fig. 1). For a wavelength of less than λ_0 the transmission curve is shifted toward the longer wavelength as a consequence of the refractive indices being higher than predicted. For a wavelength that is higher than λ_0 the transmission curve is shifted toward shorter wavelength as a consequence of the refractive indices being lower than predicted. [The materials chosen are ZnS and Misch-Flouride (trade name of mixed fluoride product of Hugo Anders, Nabburg, Germany)].

It is possible to anticipate the dispersion-induced shifting by introducing an initial correction upon the desired transmission curve that corresponds to transforming the solid curve in Fig. 1 into the dotted curve in Fig. 1.

As one can see from Eq. (2) keeping the phase constant when changing the refractive index affords a similar change in wavelength. By labeling the wavelength of the shifted points λ^*, we can express this as

$$\frac{\lambda^*}{\lambda} = \frac{n(\lambda_0)}{n(\lambda)}. \tag{14}$$

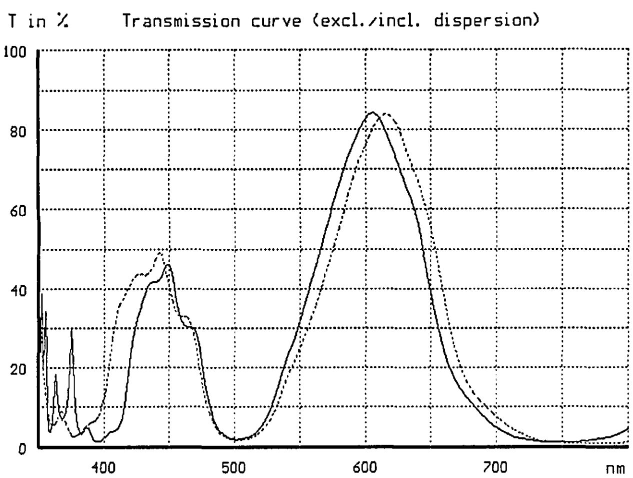

Fig. 1. Spectral performance of a GIF predicted by calculations that do not take into account dispersion (see the dotted curve). The fixed refractive indices correspond to the real ones of ZnS and Misch-Flouride at a wavelength of 500 nm. When dispersion is taken into account, the transmission curve is compressed toward the fixing point at 500 nm (see the solid curve).

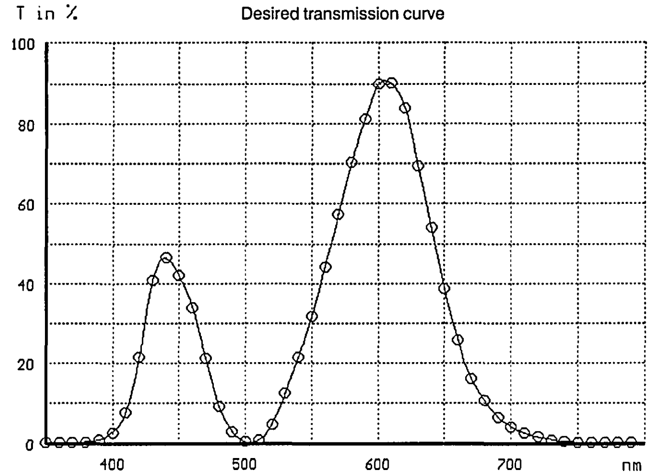

Fig. 2. Example of a chromatic filter.

In the following this phase equilibrium condition is assumed to be the main reason for the dispersion-induced shifting of the entire filter characteristic. The entire filter is constructed of alternating layers of two different dispersive materials, both contributing to the shifting. It is assumed that it is possible to estimate the shifting of the entire filter characteristic by summing two weighted contributions as expressed by

$$\frac{\lambda^*}{\lambda} = \frac{\alpha_{HL}}{1+\alpha_{HL}} \frac{n_H(\lambda_0)}{n_H(\lambda)} + \frac{1}{1+\alpha_{HL}} \frac{n_L(\lambda_0)}{n_L(\lambda)}, \tag{15}$$

where

- λ is a wavelength that corresponds to a point on the desired transmission curve,
- λ^* is the wavelength of the shifted point,
- λ_0 is the wavelength at which the refractive index is fixed when dispersion is not included (which is normally equal to λ_c),
- α_{HL} is the estimated ratio between the total optical thickness of the high-index material and the total optical thickness of the low-index material in the final filter. Normally a value of 1 is a good choice.

The efficiency of this method can be demonstrated by the following example.

Figure 2 shows an example of a chromatic filter. In the following we try to design a GIF that approaches this transmission curve as closely as possible in the wavelength range from 400 to 750 nm.

Figure 3 shows the result of the first Fourier calculation when a synthetic dispersion as described in Eqs. (12) and (13) is included. The materials selected are ZnS and Misch-Flouride. The high-index material, ZnS, becomes absorbent in the spectral range below 400 nm, a situation that is not covered by the theory. However, cutting the spectrum at 400 nm would be tantamount to affording an extremely narrow-banded rejection next to this limit, because it is assumed within the basis theory that the

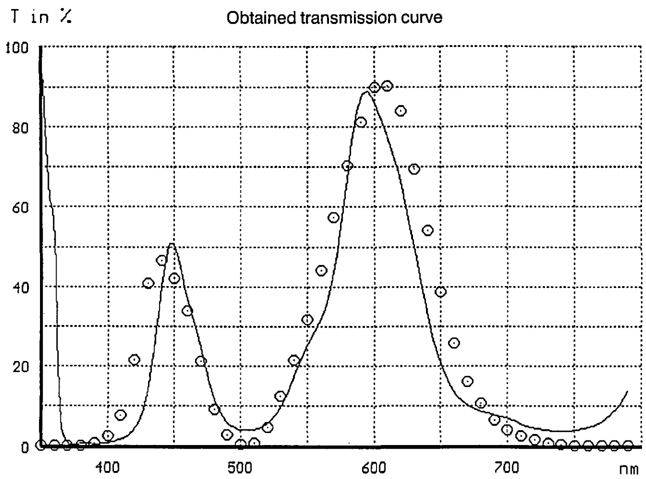

Fig. 3. Result of the first Fourier calculation when a material-dependent synthetic dispersion is included, as described in Eqs. (12) and (13). It is seen that the transmission curve is compressed around λ_0 as a result of dispersion.

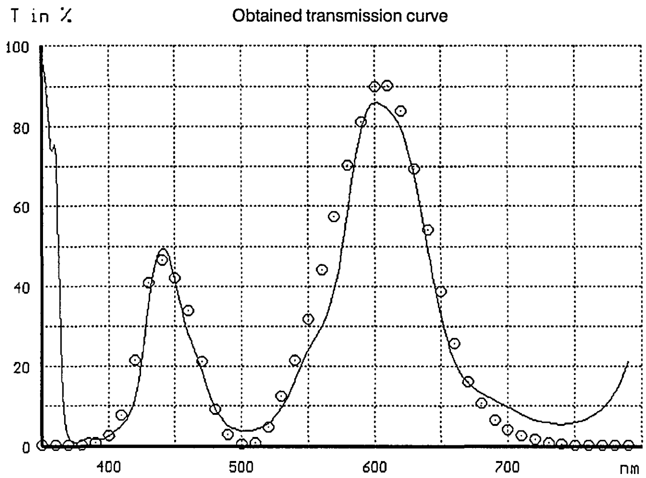

Fig. 5. Result of the first Fourier calculation when used upon the corrected desired transmission curve. When comparing Figs. 3 and 5, one sees that it is much more realistic to achieve a good solution by further refinements when the initial correction is used.

transmission is 100% outside the selected wavelength range. For this reason the low-wavelength limit is set to 350 nm, although there is no practical interest in what happens below 400 nm (see Refs. 1–7 for further details).

Figure 4 shows the new input to the calculation when the described initial correction has been performed upon the desired transmission curve ($\lambda_0 = \lambda_c = 500$ nm). Figure 5 shows the corresponding result of the first Fourier calculation when the synthetic dispersion is included. When Figs. 3 and 5 are compared, one sees that it is much more realistic to achieve a good solution by further refinements when the initial correction is used.

Designing a Real GIF

The successive approximation loop can be opened and closed again in different stages of the designing process as explained in the following.

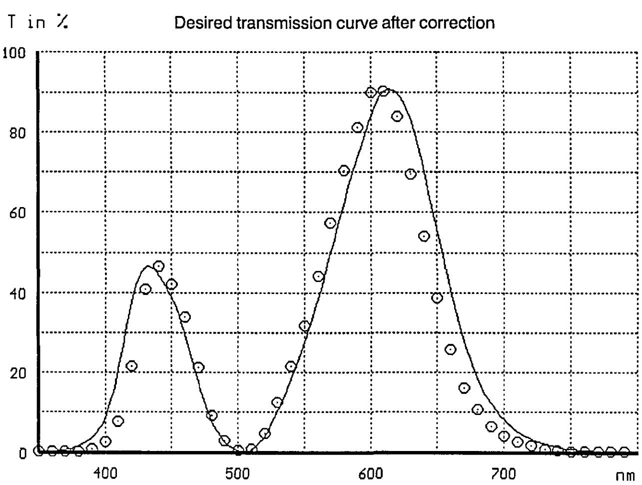

Fig. 4. New input to the calculation when the described initial correction has been performed upon the desired transmission curve in Fig. 2 ($\lambda_0 = \lambda_c = 500$ nm).

(1) Materials to be used for the production are chosen, and the ratio α_{HL} between the sum of the optical thicknesses of layers of high and low refraction in the final filter is estimated (typically it is 1).

(2) A suitable wavelength of conversion λ_c is selected (typically in the middle of the desired spectral range).

(3) Dispersion-dependent shifting in the wavelength is foreseen, and each point of the desired characteristic is moved according to Eq. (15).

(4) A suitable thickness OT_{tot} is selected on the basis of the refractive-index profile obtained.

(5) The actual spectral performance of the final filter is estimated by the inclusion of a synthetic dispersion within the calculations according to the Eqs. (12) and (13). The transmission curve obtained turns out to differ from the characteristic desired.

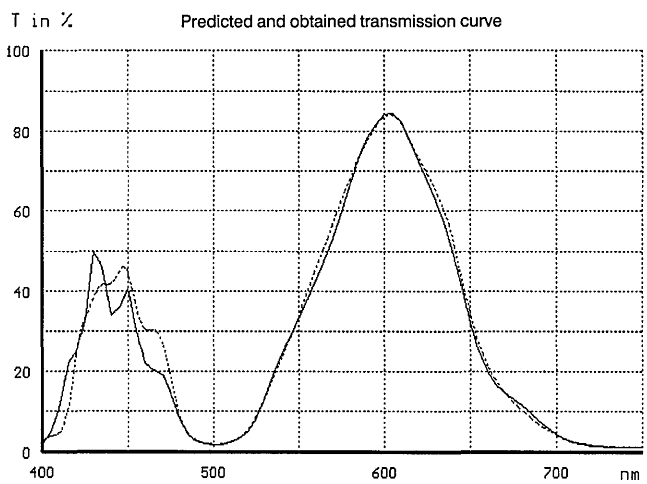

Fig. 6. Theoretically predicted spectral characteristic (dotted curve) and the obtained spectral characteristic (solid curve) of a real GIF. (The surfaces are not antireflection coated.) It is believed that deviations between the predicted and the obtained transmission curve can be reduced by further optimization of the process parameters.

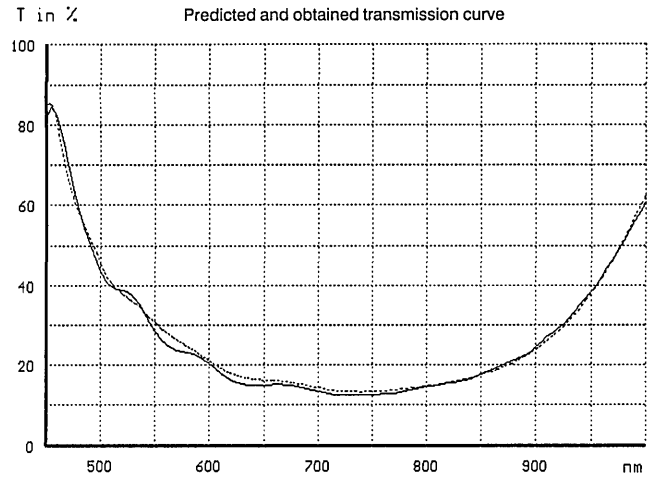

Fig. 7. Theoretically predicted spectral characteristic (dotted curve) and the obtained spectral characteristic (solid curve) of a real GIF for the linearization of a system containing a halogen lamp and a silicon photodiode. (The surfaces are not antireflection coated.)

However, because of the initial correction of the input characteristic, deviations are small.

(6) Performing a number of successive approximations and assuming that the refractive indices match at the boundaries make it possible to reduce the deviations significantly.

(7) Further initial corrections and the overlaying of quintic matching layers as explained in Ref. 7 can function at the same time easily (see Ref. 7 for further details).

(8) Converting the synthetic dispersion-corrected and optimized refractive-index profile into a two-index solution makes it possible to see the real dispersion-corrected transmission curve, which is typically quite close to the spectral characteristic desired.

(9) Closing the optimization loop again makes it possible to optimize the real transmission curve by further successive approximations. Furthermore, my system helps to eliminate unrealistic thin layers in the solution.

At Lys & Optik we have developed an advanced proprietary system for process control. This system makes it possible to control the deposition of the thin layers with good precision. Figure 6 shows the theoretical and the obtained spectral characteristic of a real GIF corresponding to the desired transmission curve in Fig. 2. It is believed that the deviations between the predicted and the achieved transmission curve can be reduced by further optimizations of the actual process control. Figure 7 shows another real GIF that was designed for the linearization of a system containing a halogen lamp and a silicon photodiode. Taking into account dispersion within the design assures us that it is possible to achieve good agreement between theory and practice in a broad spectral range. The optical thickness of the thinnest sublayers in the two examples is in the range of 10 nm.

Conclusion

The possibility of converting the continuous refractive-index profile into a two-index solution by using double-layer equivalents has been shown. The technique of conversion that was developed makes it possible to include a synthetic dispersion within the preceding calculations. A dispersion-dependent shift in wavelength is foreseen, and the possibility of obtaining solutions that are close to what was desired when initial corrections were utilized is shown. When converted, the transmittance curve obtained appears to be dispersion corrected. Two practical examples are given that demonstrate how it is possible to obtain good agreement between theory and practice in a broad spectral range.

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